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Updated g(0) estimate for western North Pacific Bryde's whales and its application to previous abundance estimates

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Abstract

An updated g(0) estimate for Bryde's whale is obtained and applied to previous abundance estimates that assumed g(0)=1. The g(0) estimates was obtained by applying mark-recapture distance sampling methods (MRDS) to sighting data from Independent Observer (IO) mode conducted during the IWC-POWER surveys in 2015 and 2016. Following suggestions from the Intersessional workshop on North Pacific Bryde's whale *Implementation Review*, a weighted harmonic mean of the g(0) estimates under good and bad Beaufort sea was obtained, by sub-areas.

INTRODUCTION

Abundance estimates for western North Pacific Bryde's whales were provided and discussed at the 2017 International Whaling Commission Scientific Committee (IWC SC) meeting (Hakamada *et al.*, 2017). Those estimates were based on g(0)=1. It is desirable to have estimates of abundance corrected for g(0) < 1. Estimates of g(0) were made by applying mark-recapture distance sampling (MRDS) methods to sighting data obtained from Independent Observer (IO) mode surveys conducted during the IWC-POWER surveys in 2015 and 2016 (Hakamada, 2018). The estimates were reviewed at the workshop on *Implementation Review* for western north Pacific Bryde's whales. The workshop agreed that similarity between dedicated sighting surveys by NRIFSF and those under JARPN II was sufficient to allow application of the IWC-POWER survey estimates of g(0) for the top barrel only (IWC, 2018). The workshop also agreed that g(0)-corrected abundance estimates for the non-POWER surveys need to take sea state into account (IWC, 2018).

This paper provides updated estimates of g(0) and their application to previous abundance estimates (Kitakado *et al.*, 2007; 2008; Hakamada *et al.*, 2017).

MATERIAL AND METHODS

Data set for g(0) estimate

There were several sightings of Bryde's whales made in IO mode during the 2015 and 2016 IWC-POWER surveys. The number of the primary sightings by duplicate status, platform and Beaufort are shown in Table 1. The number of the primary sightings of like Bryde's whale and that of large baleen whales are also shown for reference.

Species used for sensitivity test

In order to investigate sensitivity of g(0) were also obtained, results for inclusion of sighting data form other species such as 'like Bryde's whale' and 'large baleen whales' were also obtained.

Platform

There are three platforms in the research vessels; top barrel (TOP), IO platform (IOP) and upper bridge. Detections from the TOP and those from the IOP are independent (in the sense that the observers in each are unaware of sightings by the other platform), while detections from the upper bridge are not independent from detections made by the other platforms because observers on the upper bridge are informed of detections made by the TOP and the IOP. The DISTANCE package program (Thomas *et al.*, 2010) can treat double platform surveys but not triple platform ones. However, given that the number of detections from the upper bridge was only 2, detections by an observer on the upper bridge make a minor contribution only and have been excluded from this analysis which estimates g(0) for the TOP and the IOP only.

School size

As shown in Table 1, the school size is one for most of the Byrde's whale schools detected (28 out of 31). The school size is two for the rest of the Bryde's whale schools. In this situation, the dependency of detectability on school size clearly could not be estimated with any reasonable precision in this analysis. Therefore, school size has not been used as a candidate covariate for the g(0) model.

Analytical methodology

Estimate of g(0) comprises two components. One is the Mark-Recapture (MR) model and the other is distance sampling (DS) model (Hazard rate and Half normal detection functions are considered for that).

MRDS methods are described in Laake and Borchers (2004) and Burt *et al.* (2014). The analyses were conducted using library MRDS in R-DISTANCE (Thomas *et al.*, 2010).

The probability that that either or both the observers in the top barrel (1) or those in the IO platform (2) detect a school is given by

$$\hat{p}_{1\,\cup 2} = \hat{p}_{1\,|2} + \hat{p}_{2|1} - \hat{p}_{1|2}\hat{p}_{2|1}$$
(1)

It is assumed that $p_{1|2}=p_{2|1}$ for this analysis. The reasons are the limited number of data (*n*=31), together with the fact that the numbers of sightings from these two platforms are relatively similar (Table 1), which suggest that this assumption does not introduce any large error. The MR model used is described by

$$\hat{p}_{1|2}(y,z) = \hat{p}_{2|1}(y,z) = \frac{\exp(\beta_0 + \beta_1 y + \sum_{k=1}^{K} \beta_{k+1} Z_k)}{1 + \exp(\beta_0 + \beta_1 y + \sum_{k=1}^{K} \beta_{k+1} Z_k)}$$
(2)

where y is perpendicular distance from the trackline. The only covariate z considered is Beaufort sea state.

g(0) for the top barrel and IOP combined (denote $g_1 \cup_2 (0)$ hereafter) can be obtained by substituting equation (2) into equation (1). g(0) estimate for the TOP barrel only under good and bad Beaufort state are substitute relevant covariates to Equation (2).

$$g_{1,g}(0) = \hat{p}_{1|2}(0, z_g) = \frac{\exp(\beta_0 + \beta_2 z_g)}{1 + \exp(\beta_0 + \beta_2 z_g)} \quad (3a)$$
$$g_{1,b}(0) = \hat{p}_{1|2}(0, z_b) = \frac{\exp(\beta_0 + \beta_2 z_b)}{1 + \exp(\beta_0 + \beta_2 z_b)} \quad (3b)$$

where z_g and z_b are covariates for Beaufort state relevant to good and bad Beaufort state, respectively. Variance of $g_{1,g}(0)$ and $g_{1,b}(0)$ are approximated by delta method.

The DS models considered (Hazard rate and Half-normal respectively) are described by

$$f(y,z) = 1 - \exp\left[-{\frac{y}{a_0}\exp(a_1z)}\right]^{-b} \quad (4)$$

$$f(y,z) = \exp\left[-{\frac{y^2}{2a_0^2}\exp\{2(a_1z)\}}\right] \quad (5)$$

where a_0, a_1 and b are coefficients of the detection function.

Sensitivity test for g(0) estimate

Alternative scenarios were examined to investigate g(0) for the top barrel and IOP combined, g(0) for the top barrel/IOP individually ($g_1(0)$) and ESW. One scenario is for treatment of Beaufort sea state and three scenarios are for inclusion of species other than Bryde's whales. The scenarios are as follows.

- 1. Beaufort Sea State grouping:
 - 0-2: Good, 3-5: Bad (In the base case, 0-3: Good, 4-5: Bad)
- 2. Including detections of like Bryde's whales
- 3. Including detections of large baleen whales

4. Including detections of both like Bryde's whales and large baleen whales

Derivation of weighted harmonic mean of g(0)

Let abundance estimate in stratum 1 and 2 are N_1 and N_2 , respectively, assuming effective strip width w and estimated mean school size E(s) are common in two strata for the abundance estimation. N_{ave} is effort weighted average of abundance estimates in two strata assuming g(0)=1.

$$N_{ave} = \frac{L_1}{L_1 + L_2} N_1 + \frac{L_2}{L_1 + L_2} N_2$$

= $\frac{L_1}{L_1 + L_2} \frac{n_1 E(s)}{2wL_1} + \frac{L_2}{L_1 + L_2} \frac{n_2 E(s)}{2wL_2}$ (6)
= $\frac{(n_1 + n_2)E(s)}{2w(L_1 + L_2)}$

Let $g_i(0)$ be the g(0) estimate in stratum *i* and N_{ave}' is g(0)-corrected abundance for combined area, effort weighted average of abundance estimates in two strata divided by $g_i(0)$ is expressed by

$$N_{ave}' = \frac{L_1}{L_1 + L_2} \frac{N_1}{g_1(0)} + \frac{L_2}{L_1 + L_2} \frac{N_2}{g_2(0)}$$

$$= \frac{L_1}{L_1 + L_2} \frac{n_1 E(s)}{2w L_1 g_1(0)} + \frac{L_2}{L_1 + L_2} \frac{n_2 E(s)}{2w L_2 g_2(0)}$$

$$= \frac{E(s)}{2w (L_1 + L_2)} \left(\frac{n_1}{g_1(0)} + \frac{n_2}{g_2(0)}\right)$$

$$= \frac{(n_1 + n_2) E(s)}{2w (L_1 + L_2)} \frac{1}{n_1 + n_2} \left(\frac{n_1}{g_1(0)} + \frac{n_2}{g_2(0)}\right)$$

$$= N_{ave} \frac{1}{n_1 + n_2} \left(\frac{n_1}{g_1(0)} + \frac{n_2}{g_2(0)}\right)$$
(7)

Therefore, g(0) estimate for combined strata $g_{ave}(0)$ can be expressed by weighted harmonic mean of g(0) under good and bad Beaufort state;

$$\frac{1}{g_{\text{ave}}(0)} = \frac{1}{n_b + n_g} \left(\frac{n_b}{g_b(0)} + \frac{n_g}{g_g(0)} \right) \quad (8)$$

where n_g/n_b is the number of the sightings under good/bad Beaufort state. The Variance of $g_{1,ave}(0)$ is approximated by delta method.

Applying to data used for the g(0) estimate (i.e. sighting data in IO mode during 2015 and 2016 IWC-POWER surveys), $g_{1,ave}(0)$ is 0.717. Difference between $g_1(0)$ of 0.672 and $g_{1,ave}(0)$ is 0.045. The difference is subtracted from $g_{1,ave}(0)$ estimates for adjustment.

RESULTS

Model selection was conducted using AIC. Table 2 shows AIC, $g_1 \cup_2(0)$ and $g_1(0)$ estimates and their standard errors (SEs) for the models examined. The best overall model includes the MR model with the covariate Beaufort, and a DS model with no covariate. Estimated coefficients of the best model and their SEs are shown in Table 3. The Variance-covariance matrix for the coefficients is shown in Table 4. The estimates in Table 3 and 4 are used for the estimation of variance of harmonic mean $g_1(0)$ estimate. The associated $g_1(0)$ estimate for the top barrel is 0.672 (se=0.168) and the g(0) estimate for the top barrel and the IO platform combined is 0.863 (se=0.135). The g(0) estimates differ between the MR models with Beaufort as a covariate and those with no covariates. The detection function, plot of perpendicular distance and predicted probability of detection for each combination of platform are shown in Figure 1. These plots show that the fits are satisfactory. Quantile-Quantile (QQ) plot for the best model of the detection function is shown in Figure 2. The figure shows error of the model's nearly normally distributed.

For the Base Case model these $g_1 \cup_2 (0)$ and $g_1(0)$ estimates correspond to the combination of Beaufort states encountered during the cruise. The good and for bad Beaufort states, the estimates for $g_1(0)$ are 0.899 (SE=0.255) and 0.543 (SE=0.208), respectively.

Sensitivity tests for g(0) estimates

Table 5 shows that the estimates for $g_1 \cup_2(0)$, $g_1(0)$ and ESW for the AIC-selected model and for the various sensitivity tests. The $g_1 \cup_2(0)$ and $g_1(0)$ estimates differ amongst these scenarios. This is likely to be caused (in part) by small sample size (n=31) for the Base Case - note that the alternative point estimates all fall with the 95% CI for the Base Case estimate.

Abundance estimates for g(0) correction

The number of sightings under good and bad Beaufort state by years and sub-areas are shown in Table 6. Using these numbers and equation (8), weighted harmonic means of g(0) are calculated. The survey-specific g(0) estimate and their CVs are shown in Table 6.

Table 7 shows previous abundance estimates assuming g(0)=1 (Kitakado *et al.*, 2007; Kitakado *et al.*, 2008; Hakamada *et al.*, 2017). The g(0)-corrected abundance estimates and their sampling CVs are also shown in Table 7. In this table, it was assumed that the CV of the survey specific g(0) are 0.25 following to recommendation at the intersessional workshop (IWC, 2018). For comparison, abundance estimates corrected by g(0) estimate of 0.672 (CV=0.25) are also shown.

DISCUSSION

The harmonic mean of g(0) estimates for top barrel are different among sub-areas (Table 6). This dependency of the g(0) estimate on the Beaufort sea state in each sub-area is as suggested at the Workshop on RMP *Implementation Review* for the Western North Pacific Bryde's whales. This approach enables g(0)-correction of previous abundance estimates assuming g(0)=1.

The g(0)-corrected abundance estimates with sampling CV in Table 7. Abundance estimates for sub-areas 1W and 1E in 2011 were not taken unsurveyed area into account (Hakamada *et al.*, 2017). If the additional CV estimate of 0.335 (Hakamada *et al.*, 2017) is taken into account and the abundance estimates for sub-areas 1W and 1E in 2011 were extrapolated considering unsurveyed area, the estimates can be used for the trials of the *Implementation Review* for the Bryde's whales.

It was assumed that detectability at the top barrel and the IO platform are the same. The best MR models including interaction between two platforms and covariates were examined to test validity of this assumption. Table 8 shows estimated $g_{1}(0)$ and $g_{2}(0)$. The $g_{1}(0)$ and $g_{2}(0)$ estimates are not substantially different. The SE of $g_{1}(0)$ is larger than that in Table 2, though the difference is not statistically significant. Taking account of interaction between platforms and other covariates would not improve g(0) estimate and precision substantially. This suggests that this assumption is acceptable.

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Table 1. The numbers of the primary sightings by duplicate status and platform during IO mode IWC-POWER surveys in 2015 and 2016.

2015

| | Duplicate status | | Platform | | | Beaufort | | | | School size | |
|---------------------------|------------------|---|------------------|-----|----|----------|---|---|---|-------------|---|
| Species | D | Ν | ТОР | IOP | UB | 1 | 2 | 3 | 4 | 1 | 2 |
| Brydes | 5 | 6 | · 9 | 6 | 1 | 1 | 3 | 6 | 1 | 8 | 3 |
| Like . Brydes . | 0 | 1 | - - - - | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| Large baleen whales | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

2016

| | Duplicate status | | Platform | | | | Beaufort | | | | School size | |
|---------------------------|------------------|-----|----------|-----|----|---------------|----------|---|----|-------------|-------------|--|
| Species | D | Ν | TOP | IOP | UB | 1 | 2 | 3 | 4 | 1 | 2 | |
| Brydes | 13 | 7 | 18 | 15 | 0 | 0 | 5 | 4 | 11 | 20 | 0 | |
| Like Brydes | 0 | 4 - | 1 | 2 | 1 | - - 0 - | 2 | 2 | 0 | - 4 | 0 | |
| Large baleen whales | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 1 1 | 0 | |

Platform TOP: Top barrel, IOP: IO platform, UB: Upper bridge

Duplicate status D: Definite duplicate, N: Not duplicate

Table 2. The g(0) estimates with SEs for the models considered. $g_1 \cup_2 (0)$ is the estimate for the top barrel and IO platform. $g_1(0)$ is the estimate for the top barrel only. B refers to the Beaufort covariate. The best model as indicated by AIC is shown in **bold**.

| MR model | DS model | Functio Form | | | | | | |
|-------------|-------------|-----------------|--------|--------|------------------------|-----------------|----------|--------------|
| Covs | Covs | DS | AIC_MR | AIC_DS | $g_1 U_2\left(0 ight)$ | $se(g_1U_2(0))$ | $g_1(0)$ | $se(g_1(0))$ |
| None | None | HN | 59.903 | 60.616 | 0.945 | 0.055 | 0.765 | 0.118 |
| None | None | HR | 59.903 | 60.687 | 0.945 | 0.055 | 0.765 | 0.118 |
| None | В | HN | 59.903 | 61.932 | 0.945 | 0.055 | 0.765 | 0.118 |
| None | В | HR | 59.903 | 62.196 | 0.945 | 0.055 | 0.765 | 0.118 |
| В | None | HN | 56.426 | 60.616 | 0.863 | 0.135 | 0.672 | 0.168 |
| В | None | HR | 56.426 | 60.687 | 0.863 | 0.135 | 0.672 | 0.168 |
| В | В | HN | 56.426 | 61.932 | 0.876 | 0.125 | 0.695 | 0.161 |
| В | В | HR | 56.426 | 62.196 | 0.875 | 0.127 | 0.693 | 0.164 |

Table 3. Estimates of the coefficients and their standard errors (SE) for the best g(0) model.

| Covariate | Estimate | SE |
|-----------|----------|-------|
| Intercept | -1.839 | 1.537 |
| distance | 0.132 | 0.551 |
| Beaufort | 2.013 | 0.942 |

Table 4. Variance-covariance matrix of estimated coefficients for the best g(0) model.

| | Intercept | distance | Beaufort |
|-----------|-----------|----------|----------|
| Intercept | 2.364 | -0.490 | -1.274 |
| distance | -0.490 | 0.304 | 0.116 |
| Beaufort | -1.274 | 0.116 | 0.888 |

Table 5. Sensitivity test for g(0) estimate for the best model. $g_{1\cup 2}(0)$ is g(0) for TOP barrel and IO platform and $g_1(0)$ is g(0) for TOP barrel.

| Scenario | MR model Covs | DS model Covs | Function Form for DS | $g_1 U_2(0)$ | $\operatorname{se}(g_1 \cup_2 (0))$ | $g_1(0)$ | $se(g_1(0))$ |
|----------------------|---------------------|---------------------|----------------------------|--------------|-------------------------------------|----------|--------------|
| Base case | В | None | HN | 0.863 | 0.135 | 0.672 | 0.168 |
| Beaufort grouping | В | В | HR | 0.79 | 0.247 | 0.612 | 0.245 |
| +Like Bryde's | В | None | HN | 0.781 | 0.178 | 0.577 | 0.183 |
| +Large baleen whales | В | None | HN | 0.922 | 0.080 | 0.749 | 0.131 |
| +Both species | В | None | HN | 0.865 | 0.117 | 0.668 | 0.150 |

| Year | Sub-area | Good | Bad | $g_{1,ave}(0)$ | CV |
|------|----------|------|-----|----------------|-------|
| | 1W | 174 | 111 | 0.671 | 0.239 |
| 1995 | 1E | 90 | 47 | 0.689 | 0.227 |
| | 2 | 22 | 16 | 0.659 | 0.278 |
| | 1W | 76 | 28 | 0.719 | 0.203 |
| 2000 | 1E | 58 | 109 | 0.584 | 0.328 |
| | 2 | 15 | 6 | 0.712 | 0.259 |
| 2011 | 1W | 73 | 92 | 0.613 | 0.299 |
| 2011 | 1E | 81 | 29 | 0.721 | 0.200 |
| 2014 | 2 | 30 | 27 | 0.641 | 0.285 |

Table 6. The number of the primary sightings of the Bryde's whale by good/bad Beaufort state and estimated average $g_{1,ave}(0)$ (i.e. Equation(8)) and their CVs.

Table 7. Previous abundance estimate assuming g(0)=1 and g(0)-corrected abundance estimate using g(0) estimate in Table 6 and g(0)=0.672 (CV=0.25). It is assumed that CV of the survey specific g(0) is 0.25.

| Year | Sub-area | g(0)=1 | | g(0) in | Table 6 | g(0)=0.672 | |
|------|----------|--------|------------------|---------|------------------|------------|------------------|
| | Sub-area | Р | $\mathrm{CV}(P)$ | Р | $\mathrm{CV}(P)$ | Р | $\mathrm{CV}(P)$ |
| | 1W | 8,152 | 0.329 | 12,149 | 0.413 | 12,131 | 0.413 |
| 1995 | 1E | 10,814 | 0.342 | 15,695 | 0.424 | 16,092 | 0.424 |
| | 2 | 2,860 | 0.372 | 4,340 | 0.448 | 4,256 | 0.448 |
| | 1W | 4,957 | 0.398 | 6,894 | 0.470 | 7,376 | 0.470 |
| 2000 | 1E | 11,213 | 0.498 | 19,200 | 0.557 | 16,686 | 0.557 |
| | 2 | 4,331 | 0.553 | 6,083 | 0.607 | 6,445 | 0.607 |
| 2011 | 1W | 15,422 | 0.289 | 25,158 | 0.382 | 22,949 | 0.382 |
| | 1E | 6,716 | 0.216 | 9,315 | 0.330 | 9,994 | 0.330 |
| 2014 | 2 | 4,161 | 0.264 | 6,491 | 0.364 | 6,192 | 0.364 |

Table 8. g(0) estimates and their se's for TOP barrel ($g_1(0)$) and IO platform ($g_2(0)$).

| $g_1(0)$ | $se(g_1(0))$ | $g_2(0)$ | $se(g_2(0))$ |
|----------|--------------|----------|--------------|
| 0.654 | 0.227 | 0.680 | 0.188 |

Observer = 1 detections

Observer = 2 detections

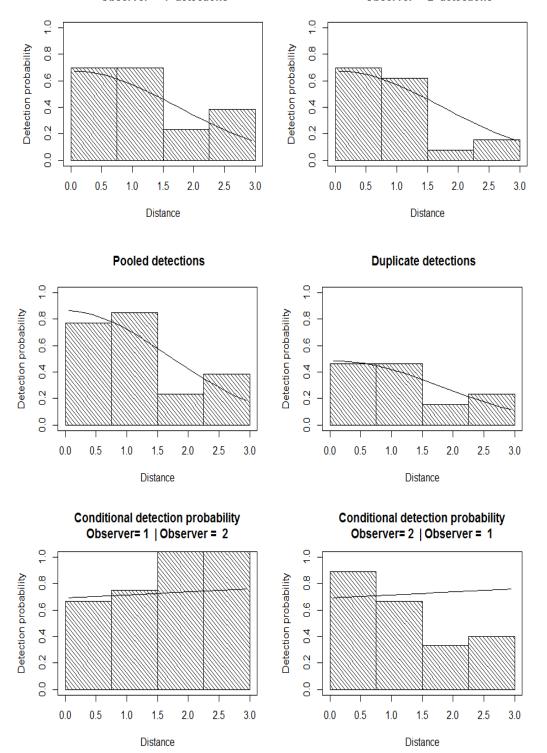


Figure 1. Detection function of the best model for top barrel (Observer = 1) (upper left panel), that for IO platform (Observer =2) (upper right panel), that for observers on either top barrel or IO platform (middle left panel), that estimated from observations by both platforms on top barrel and IOP (middle right panel), that for observers on top barrel under the condition that school is detected by the observer on IOP (lower left panel) and that for observers on IOP under the condition that school is detected

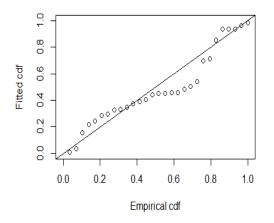


Figure 2. Quantile-Quantile (QQ) plot for detection function of the best model. X axis is empirical cumulative distribution function (cdf) of normal distribution. Y axis is error of the detection function model. Left panel is QQ plot for the best model and right panel is that for interaction term are added to the best model.