A Note of a Posterior Distribution for r_0 for Baleen Whale Stocks at Low Abundance

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ABSTRACT
A Bayesian hierarchical meta-analysis is conducted using the data for rates of increase for 13 baleen whale stocks at low population size. The posterior predictive distributions for r_0 by stock are generally centered on the point estimates of r_0 for most stocks, unlike when a posterior is developed for r_0/r_{max}. The lower 5% and 10% points of the posterior predictive distributions for r_0 for an unknown stock are 0.029 and 0.037 respectively.

KEYWORDS: MSYR; META-ANALYSIS

INTRODUCTION

The 78th intersessional workshop on the review of maximum sustainable yield rates in baleen whales (IWC, 2013) conducted a Bayesian hierarchical meta-analysis for the quantity r_0/r_{max}, i.e. the ratio of the increase rate in the limit of zero population size to the maximum rate of increase of a whale stock which is demographically possible. The workshop noted that posterior for this quantity is bounded above by 1, which means that the bulk of the mass of the posterior predictive distributions for r_0 for some stocks will lie below the point estimate (Fig. 1 of IWC[2013]). IWC (2013) therefore recommended that a meta-analysis of r_0 values, ignoring the impact of environmental variation on r_0, be conducted. The author of this paper reluctantly agreed to do this.

METHODS

Data utilized

The estimates of r_0 and measures of their uncertainty were those selected by IWC (2013) [Table 1].

Analysis method

The estimates of r_0 are assumed to be generated from a beta distribution with limits 0 and 0.2 (IWC, 2013), and the estimates for each stock are assumed to be normally distributed with mean and standard deviation for each stock given in Table 1. The marginal likelihood for the data given the values for the hyper-parameters is:

$$L(D\,|\,\alpha,\beta)=\prod_i\int_0^1\frac{\Gamma(\alpha,\beta)}{\Gamma(\alpha)\Gamma(\beta)}\chi_i^{\alpha-1}(1-\chi_i)^{\beta-1}\frac{1}{\sqrt{2\pi}\sigma_i}e^{-[\hat{r}_i-0.2\chi_i]^2/(2\sigma_i^2)}d\chi_i \qquad (1)$$

where $\hat{r}_i$ is the estimate of the rate of increase for stock i, and σ_i is the sampling standard error for $\hat{r}_i$. The hyper-priors for α and β are U[0,3.3], as specified by IWC (2013).

RESULTS AND DISCUSSION

Figure 1 shows the sampling distributions for r_0 (based on the estimates in Table 1), along with the posteriors for r_0 based on the meta-analysis. There is no evidence for major truncation effects, unlike Fig. 2 of IWC (2013) because the $r_0 \leq r_{max}$ constraint is not imposed in the analyses of this paper. The posterior distributions for r_0 for fin whales off northern

Norway and humpback whales off western Australia are truncated at r_0=0.2, while the posterior distributions for r_0 for fin whales off northern Norway and in the eastern North Pacific are truncated at r_0=0 (Fig. 1).

Figure 2 shows the posterior distribution for r_0 for an unknown stock, expressed as a probability density function and as a cumulative probability distribution, as well as the posterior distributions for $r_0 / \hat{r}_0$, the ratio of the rate of increase relative the observed rate of increase, for each stock. As expected from Figure 1, the medians of the posteriors for r_0 generally match the point estimates of r_0 with two obvious exceptions. In particular, the median for the posterior for r_0 for north Norway fin whales is markedly higher than the point estimate. This occurs because a substantial fraction of the sampling distribution for r_0 for this stock is < 0.

The lower 5% and 10% points of the posterior predictive distributions for r_0 for an unknown stock are 0.029 and 0.037 respectively.

References

IWC. 2013. Report of the intersessional workshop on the review of maximum sustainable yield rates (MSYR) in baleen whales. IWC Document SC/65A/Rep3. 14pp.

Table 1

Estimates of rates of increase used as r_0 and the associated time periods over which they were estimated (reproduced from IWC [2013]).

	r_0 (%) (95% CI)	SE	Time period	Year-span
Blue whale				
Central N Atlantic	9.0 (2.0, 17.0)	3.83	1987-2001	15
S Hemisphere	8.2 (1.6, 14.8)	3.37	1978/9-2003/4	26
EN Pacific	3.2	1.4	1991-2005	16
Fin whale				
N Norway	5 (-13, 26)	9.95	1988-98	11
EN Pacific	4.8 (-1.6, 11.1)	3.24	1987-2003	15
Humpback whale				
W Australia	10.1 (0.9, 19.3)	4.69	1982-94	13
E Australia	10.9 (10.5, 11.4)	0.23	1984-2007	24
EN Pacific	6.4	0.9	1992-2003	12
Hawaii	10 (3-16)	3.32	1993-2000	18
Bowhead whale				
B-C-B	3.9 (2.2, 5.5)	0.84	1978-2001	24
Southern right whale				
SE Atlantic (S Africa)	6.8 (6.4, 7.2)	0.2	1979-2010	32
SW Atlantic (Argentina)	6.0 (5.5, 6.6)	0.28	1971-2010	40
SE Indian (Australia)	6.6 (3.8, 9.3)	1.40	1993-2010	18

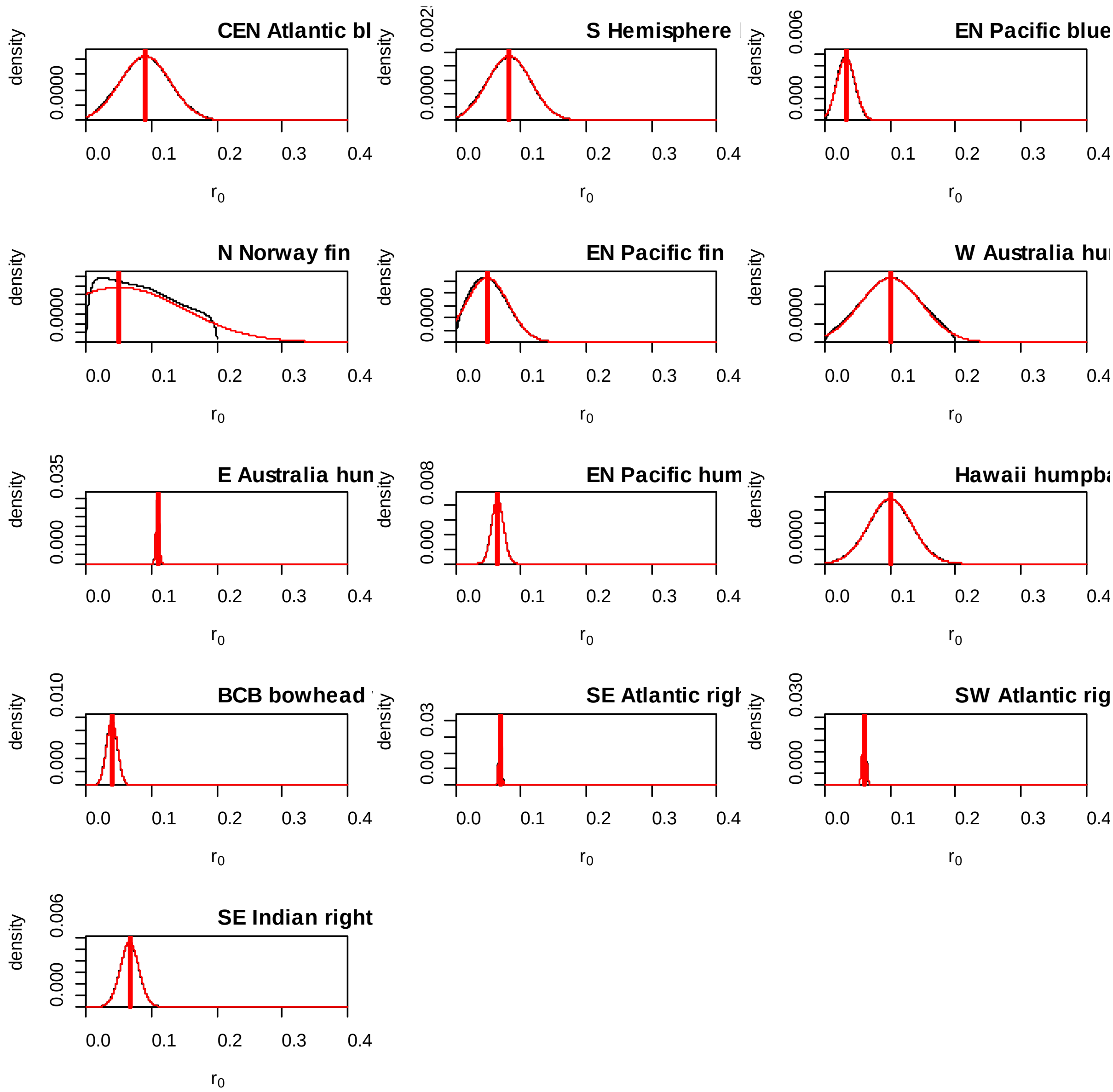

Fig. 1. Sampling distributions for r_0 (red distributions and the vertical red lines) and the posterior predictive distributions for r_0 (black distributions) by stock.

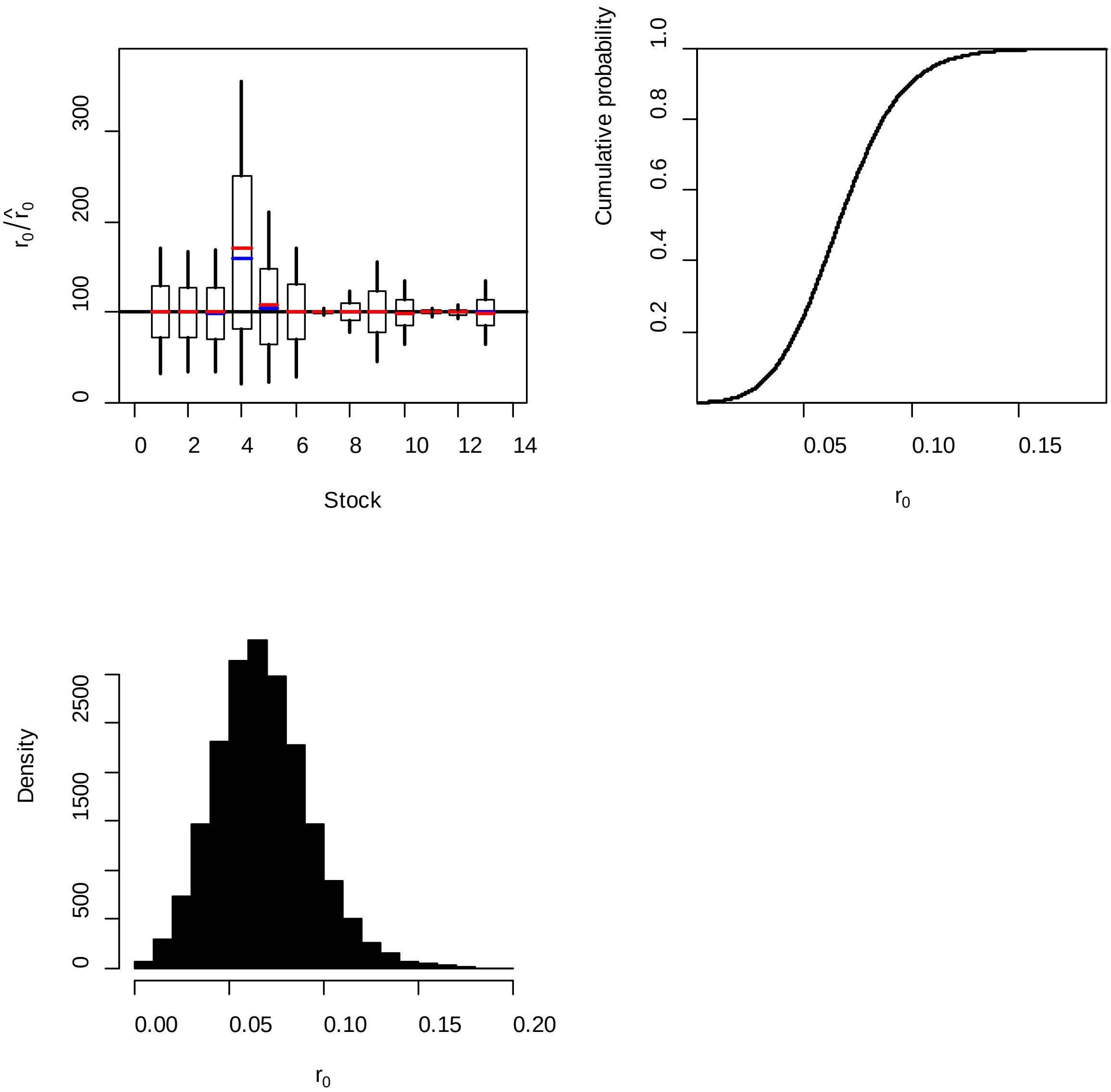

Fig. 2. Posterior distributions for the ratio of r_0 to the observed rate of increase (upper left panel), the cumulative posterior distribution for r_0 (upper right panel), and the posterior distribution for r_0 (lower left panel).