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An Additional Sensitivity Test for the r_0/r_{max} Meta-Analysis

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ABSTRACT
A Bayesian hierarchical meta-analysis is conducted using the data for rates of increase for 13 baleen whale stocks at low population size. The extent of environmental variation in r_0 as a function of r_0/r_{max} is determined from a model developed by de la Mare rather than that of Cooke. The lower 5% and 10% points of the posterior predictive distribution for r_0/r_{max} for an unknown stock are 0.419 and 0.512 respectively.

KEYWORDS: MSYR; META-ANALYSIS

INTRODUCTION

The 78th intersessional workshop on the review of maximum sustainable yield rates in baleen whales (IWC, 2013) recommended that the sensitivity of meta-analysis results be explored to the impact of variance in death as well as birth rates. De la Mare (2013) developed a relationship between r_0/r_{max} for the CV for the rate of increase at low stock size, i.e. $CV(r_0) = -0.0441(r_0 / r_{max} - 1)$ based on a model in which food availability drives variation in births and deaths. This paper conducts a Bayesian meta-analysis to develop a probability distribution for r_0/r_{max} for an unknown stock, in which extent of environmental variation in r_0 as a function of r_0/r_{max} is determined from the model developed by de la Mare rather than that of Cooke, which forms the reference case for the meta-analysis (IWC, 2013).

METHODS

Data utilized

The estimates of r_0 and measures of their uncertainty were those selected by IWC (2013) [Table 1].

Analysis method

The estimates of r_0/r_{max} are assumed to be generated from a beta distribution with limits 0 and 1 (IWC, 2013), and the estimates of r_0 for each stock are assumed to be normally distributed. The marginal likelihood for the data given the values for the hyper-parameters is:

$$L(D \mid \alpha, \beta) = \prod_i \int_0^1 \frac{\Gamma(\alpha,\beta)}{\Gamma(\alpha)\Gamma(\beta)} \chi_i^{\alpha-1}(1-\chi_i)^{\beta-1} \frac{1}{\sqrt{2\pi}\tilde{\sigma}_i} e^{-[\hat{r}_i - \chi_i r_{max}^i]^2/(2\tilde{\sigma}_i^2)} d\chi_i \quad (1)$$

where $\hat{r}_i$ is the estimate of the rate of increase for stock i, r_{max}^i is r_{max} for stock i, $\tilde{\sigma}_i^2$ is the total variance:

$$\tilde{\sigma}_i^2 = \{0.0441(1-\chi_i) r_{max}^i\}^2 + \sigma_i^2 \quad (2)$$

and σ_i is the sampling standard error for $\hat{r}_i$. The hyper-priors for α and β are U[0,3.3], as specified by IWC (2013).

RESULTS AND DISCUSSION

Figure 1 shows the sampling distributions for r_0 (based on the estimates in Table 1), along with the posteriors for r_0 based on the meta-analysis. Figure 2 shows the posterior distribution for r_0/r_{max} for an unknown stock, expressed as a probability density function and as a cumulative probability distribution, as well as the posterior distributions for $r_0 / \hat{r}_0$, the ratio of the rate of increase relative the observed rate of increase, for each stock.

The lower 5% and 10% points of the posterior predictive distributions for r_0/r_{max} for an unknown stock are 0.419 and 0.512 respectively. These values are slightly higher than the values for the reference case analysis in IWC (2013) which were 0.396 and 0.490 respectively

References

De la Mare, W.K. 2013. A note on variability in R0 calculated from an individually based baleen whale energetic model. IWC Document SC/65a/RMPx xxpp.

IWC. 2013. Report of the intersessional workshop on the review of maximum sustainable yield rates (MSYR) in baleen whales. IWC Document SC/65a/Rep5. 14pp.

Table 1
Estimates of rates of increase used as r_0 and the associated time periods over which they were estimated (reproduced from IWC [2013]).

	r_0 (%) (95% CI)	SE	Time period	Year-span	r_{max}
Blue whale					
Central N Atlantic	9.0 (2.0, 17.0)	3.83	1987-2001	15	0.114
S Hemisphere	8.2 (1.6, 14.8)	3.37	1978/9-2003/4	26	0.114
EN Pacific	3.2	1.4	1991-2005	16	0.114
Fin whale					
N Norway	5 (-13, 26)	9.95	1988-98	11	0.114
EN Pacific	4.8 (-1.6, 11.1)	3.24	1987-2003	15	0.114
Humpback whale					
W Australia	10.1 (0.9, 19.3)	4.69	1982-94	13	0.103
E Australia	10.9 (10.5, 11.4)	0.23	1984-2007	24	0.103
EN Pacific	6.4	0.9	1992-2003	12	0.103
Hawaii	10 (3-16)	3.32	1993-2000	18	0.103
Bowhead whale					
B-C-B	3.9 (2.2, 5.5)	0.84	1978-2001	24	0.043
Southern right whale					
SE Atlantic (S Africa)	6.8 (6.4, 7.2)	0.2	1979-2010	32	0.076
SW Atlantic (Argentina)	6.0 (5.5, 6.6)	0.28	1971-2010	40	0.076
SE Indian (Australia)	6.6 (3.8, 9.3)	1.40	1993-2010	18	0.076

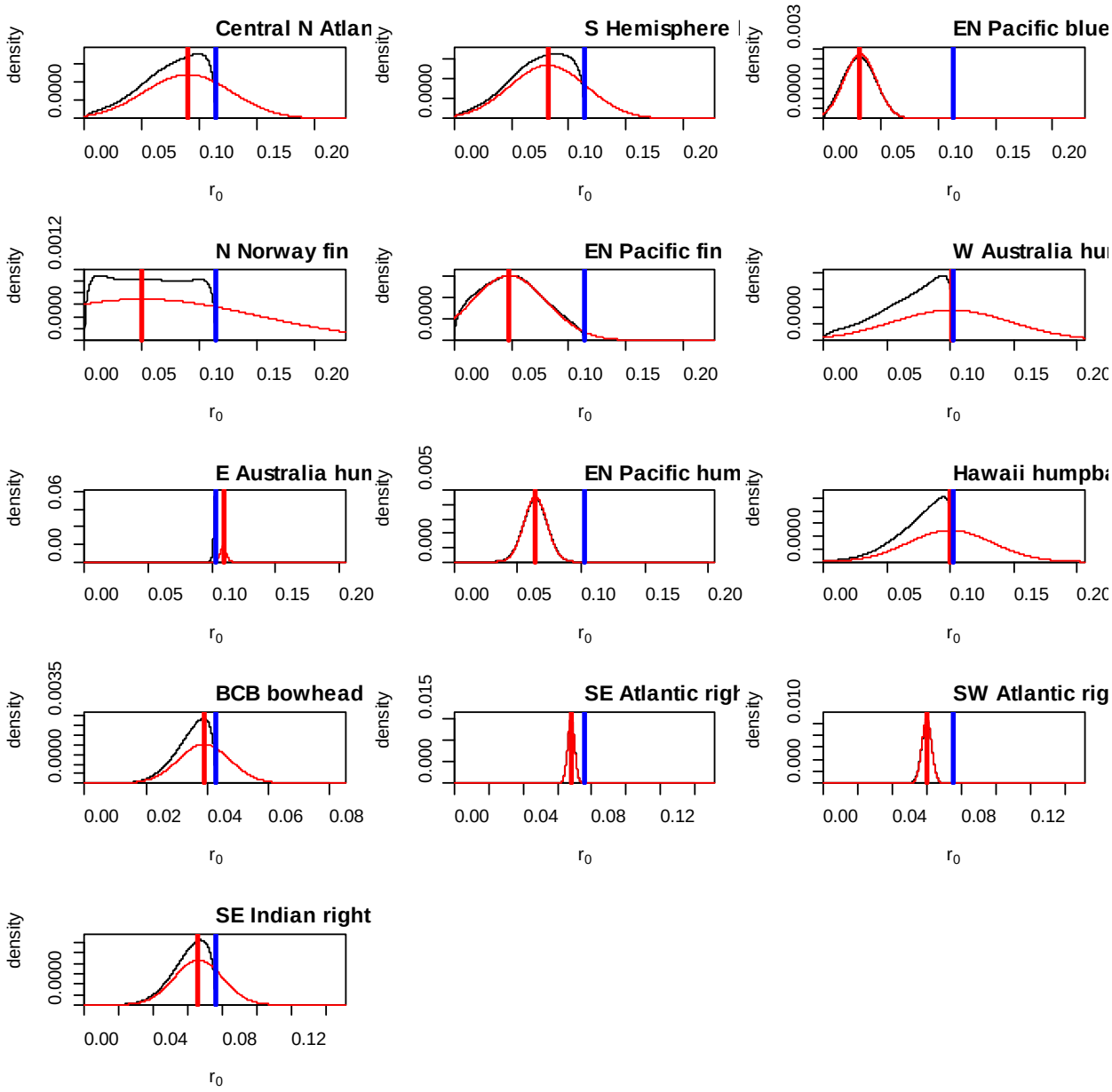

Fig. 1. Sampling distributions for r_0 (red distributions and the vertical red lines) and the posterior predictive distributions for r_0 (black distributions) by stock. The blue vertical line indicates r_{max}.

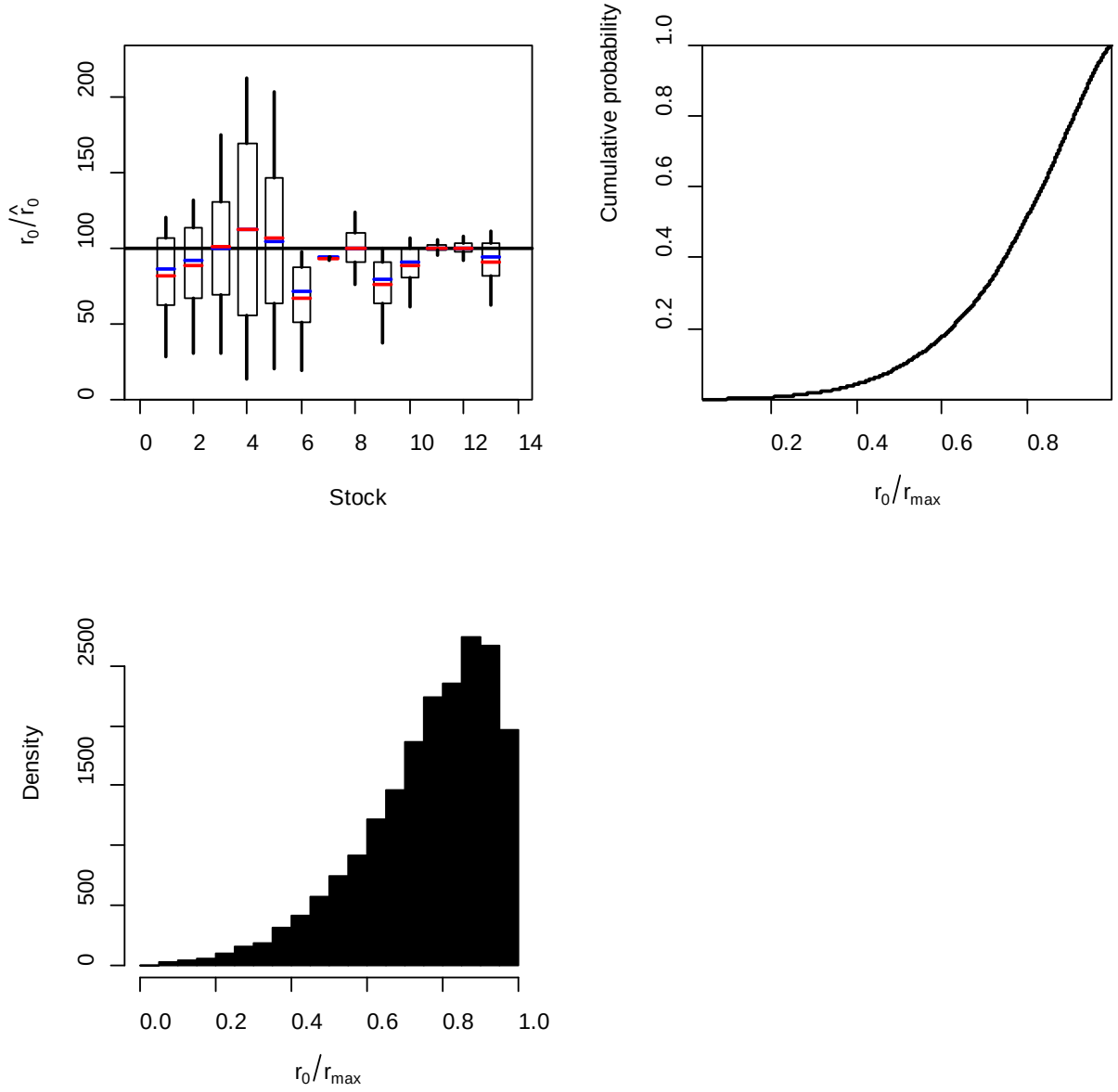

Fig. 2. Posterior distributions for the ratio of r_0 to the observed rate of increase (upper left panel), the cumulative posterior distribution for $r_{0/}r_{max}$ (upper right panel), and the posterior distribution for $r_{0/}r_{max}$ (lower left panel).